St. Agnes RC Primary School

Calculation Policy

September 2019

Progression towards a standard written method of calculation

Introduction

This calculation policy has been written in line with the programmes of study taken from the revised National Curriculum for Mathematics (2014). The aim is that children will use mental methods or jottings when appropriate, but for calculations that they cannot do in their heads they should use an efficient written method accurately and with confidence. *It is important that children's mental methods of calculation are practised and secured* ***alongside*** *their learning and use of written methods.*

Why do we need this policy?

This policy sets out those written methods which will be taught throughout St Agnes' Primary school.
It is important that the methods taught are consistent throughout the school, that there is progression from informal/practical methods of recording to written methods for each of the four operations and that parents understand what their children are learning.

Practical equipment to be used when teaching

- ***Real objects where appropriate***
- ***Numicon***
- ***Unifix cubes***
- ***Straws (whole numbers and decimals)***
- ***Base 10 equipment (deines blocks)***
- ***Place value counters***
- ***Cuisenaire rods***

Things to remember

- Children should always develop a solid understanding of place value prior to any work on the four operations – numicon, unifix, straws, base 10/deines blocks, Cuisenaire rods, money and place value counters will be used to support this.

- Practical equipment will be used when introducing, reviewing a calculation method or extending to more difficult numbers.

- Pay attention to mathematical language. When discussing any operation, the terms number sentence or calculation will be used, not sum as this can lead to confusion when not teaching addition.

- Teaching to mastery (concrete → pictorial → abstract)

- Children should always be taught to estimate first – this will help them to know if their answer is reasonable.

- Children should be encouraged to always check their answer, preferably using a different method e.g the inverse operation

- Children who make persistent mistakes should return to using practical equipment to ensure the correct method is being used.

- Children need to know number and multiplication facts by heart to ensure they are using the written method efficiently.

- Once children have an understanding of the method of calculation, opportunities will be given for children to apply what they have learned, through reasoning and problem solving activities.

- CARRY DIGITS WILL BE RECORDED AT THE TOP OF THE CALCULATION.

Progression Towards a Written Method for Addition

In developing a written method for addition, it is important that children understand the concept of addition, in that it is:

- Combining two or more groups to give a total or sum
- Increasing an amount

They also need to understand and work with certain principles, i.e. that it is:

- the inverse of subtraction
- commutative i.e. 5 + 3 = 3 + 5
- associative i.e. 5 + 3 + 7 = 5 + (3 + 7)= (5+3)+7

The fact that it is commutative and associative means that calculations can be rearranged, e.g. 4 + 13 = 17 is the same as 13 + 4 = 17.

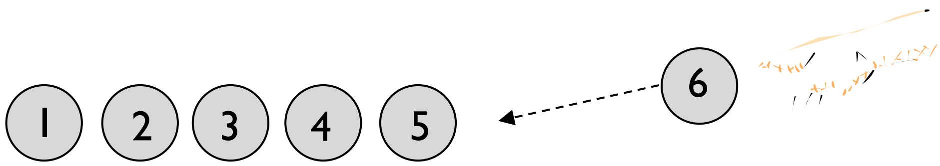
Reception

It is very important that children develop a good understanding of the number system and how it works. Children need to develop 'number sense' – to know what '5' is. This should be developed alongside their experiences of addition. Practical opportunities should be provided using a wide variety of equipment, including small world play, role play, counters, cubes etc. Key vocabulary should be modelled and used during as many real life situations as possible. It is also important that the initial building blocks for mathematical thinking, reasoning and problem solving are also encouraged.

Counting all method

Children will begin to develop their ability to add by using practical equipment to count out the correct amount for each number in the calculation and then combine them to find the total. For example, when calculating 4 + 2, they are encouraged to count out four counters and count out two counters.

To find how many altogether, touch and drag them into a line one at a time whilst counting.

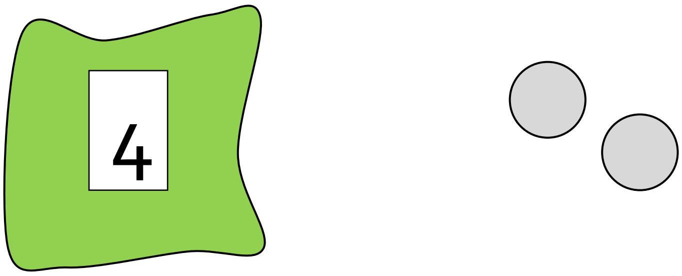


By touch counting and dragging in this way, it allows children to keep track of what they have already counted to ensure they don't count the same item twice.

Counting on method

To support children in moving from a counting all strategy to one involving counting on, children should still have two groups of objects but one should be covered so that it cannot be counted. For example, when calculating 4 + 2, count out the two groups of counters as before,

then cover up the larger group with a cloth.



For most children, it is beneficial to place the digit card on top of the cloth to remind the children of the number of counters underneath. They can then start their count at 4, and touch count 5 and 6 in the same way as before, rather than having to count all of the counters separately as before.

Those who are ready may record their own calculations.

Year One

Children will be introduced to formal recording of number sentences e.g 3+5=8. They will continue to use practical equipment, combining groups of objects to find the total by counting all or counting on. Using their developing understanding of place value, they will move on to use straws or Base 10 equipment to make teens numbers using separate tens and ones. Children will need a solid understanding that a 10 rod is the same as 10 ones.

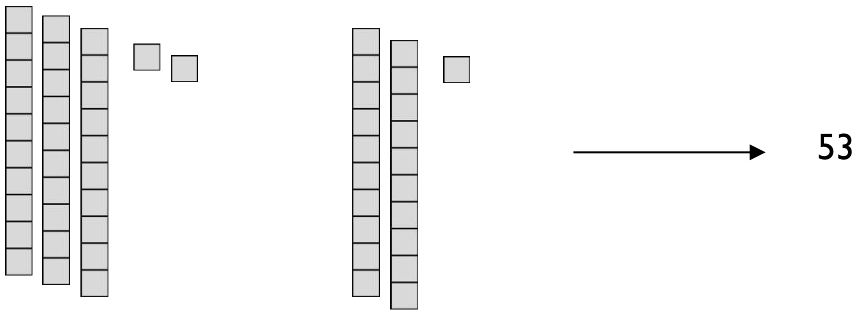
For example, when adding 11 and 5, they can make the 11 using a ten rod and a one.

The ones can then be combined to help see the final total, e.g.

so 11 + 5 = 16. They will use two different colours of base 10 equipment so that the initial amounts can be seen.

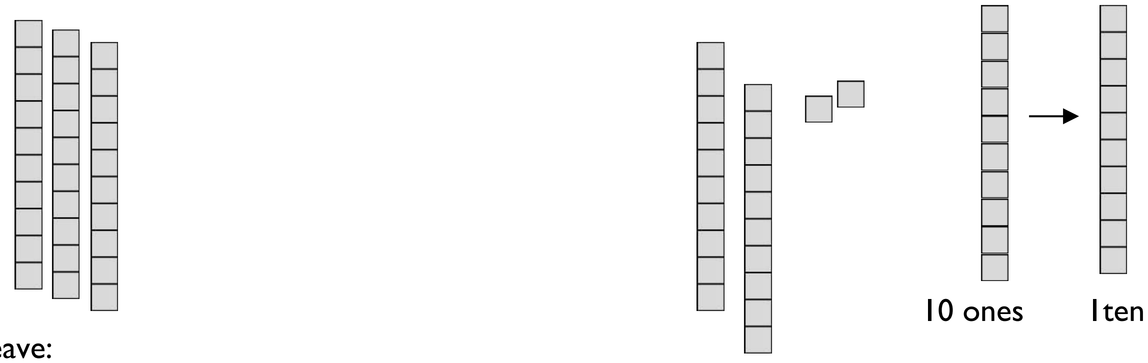
Year Two

Children will continue to use straws or Base 10 equipment to support their calculations. For example, to calculate 32 + 21, they make the individual amounts, combine and count the ones first and then combine and count the tens.

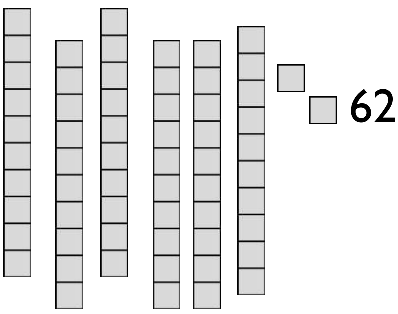


When the ones total more than 10, children should be encouraged to exchange 10 ones for 1 ten. This is the start of children understanding exchanging in vertical addition. For example, when calculating 35 + 27, they can represent the amounts using Base 10 as shown:

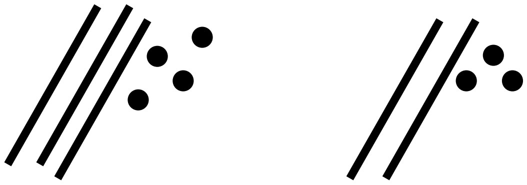
Then, identifying the fact that there are enough ones to exchange for a ten, they can carry out this exchange:



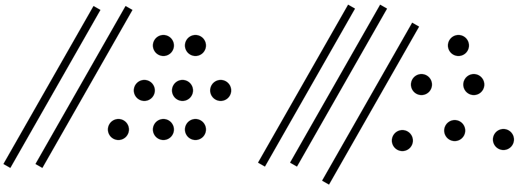
To leave:



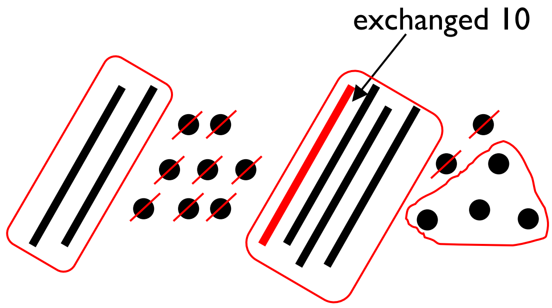
Children can also record the calculations using their own drawings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the one blocks).
e.g. 34 + 23 =



With exchange:
e.g. 28 + 36 =



will become



so 28 + 36 = 64

It is important that children circle the remaining tens and ones after exchange to identify the amount remaining.

Year Three

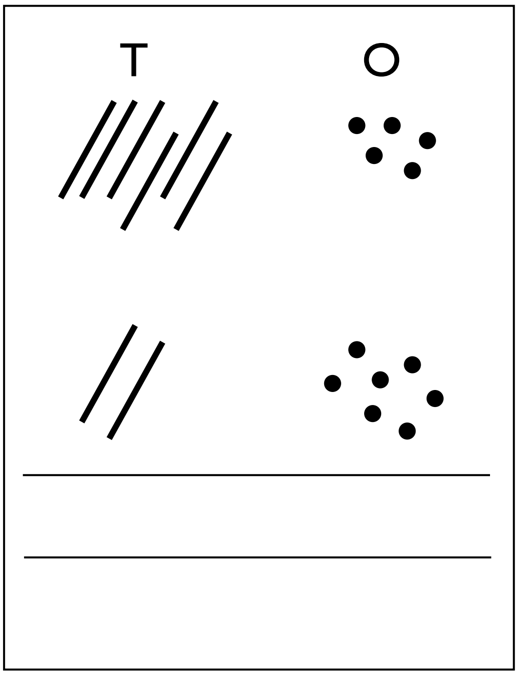
Children will build on their knowledge of using Base 10 equipment from Y2 and continue to use the idea of exchange.

Children should add the ones first, and in an identical method to that from year 2, should identify whether there are greater than ten ones which can be exchanged for one ten.

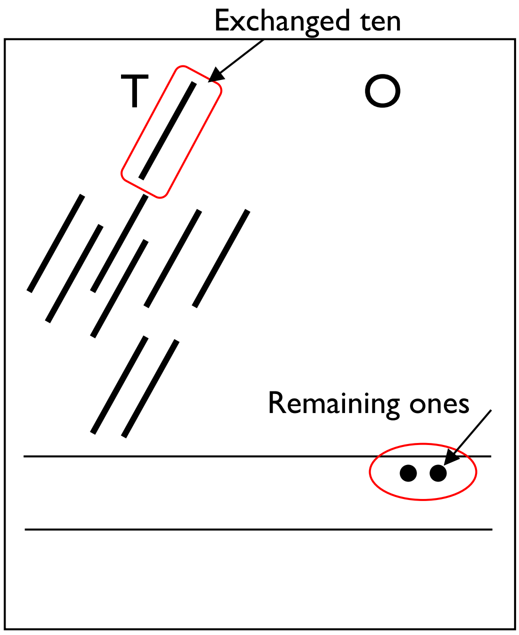
They will use a place value grid to begin to set the calculation out vertically, initially using Base 10, then using drawings, to support their knowledge of exchange between columns (as in Step 1 in the diagram below).

e.g. 65 + 27

Step 1



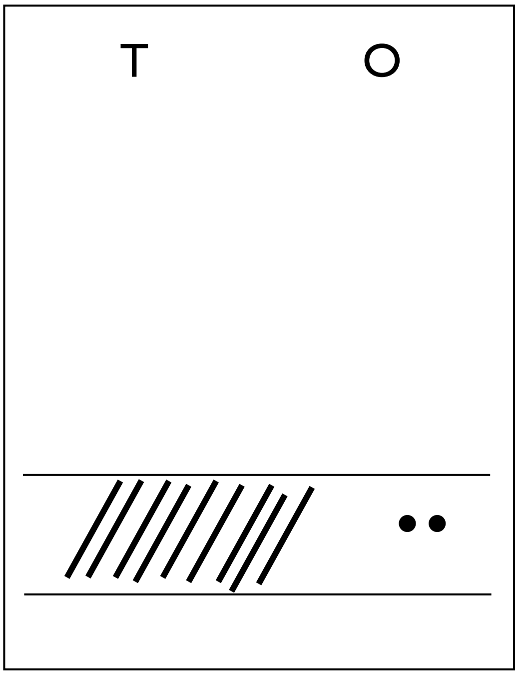
Step 2



Children would exchange ten ones for a ten, placing the exchanged ten above the equals sign. Any remaining ones that cannot be exchanged for a ten move into the equals sign as they are the ones part of the answer (as in the diagram in Step 2 above).

If there are any tens that can be exchanged for one hundred, this can be done next. If not, the tens move into the equals sign as they are the tens part of the answer (as in the diagram in Step 3 below).

Step 3



Written method

Step 1

$$\begin{array}{rcc} & T & O \\ & 6 & 5 \\ + & 2 & 7 \\ \hline & & \\ \hline \end{array}$$

Step 2

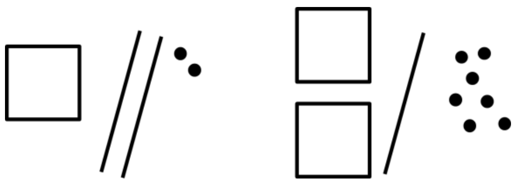
$$\begin{array}{rcc} & T & O \\ & 1 & \\ & 6 & 5 \\ + & 2 & 7 \\ \hline & & 2 \\ \hline \end{array}$$

Step 3

$$\begin{array}{rcc} & T & O \\ & 1 & \\ & 6 & 5 \\ + & 2 & 7 \\ \hline & 9 & 2 \\ \hline \end{array}$$

Children should utilise this practical method to link their understanding of exchange to how the column method is set out. Teachers should model the written method alongside the practical method. This should progress to children using the written and practical methods alongside each other and finally, when they are ready, to children using just the written method.

By the end of year 3, this method can also be used when adding three digit numbers, e.g. 122 + 217 using a square as the representation of 100.

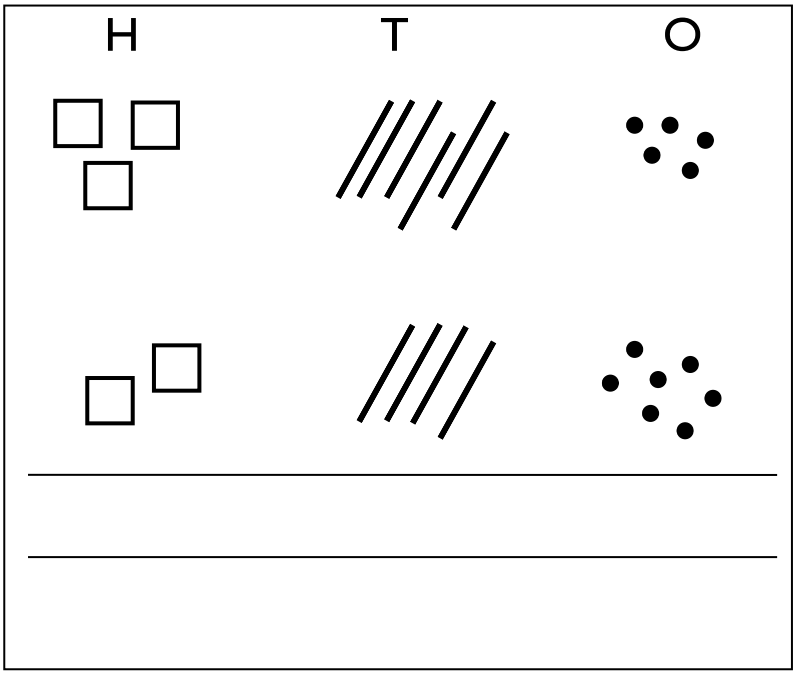


Year Four

Children will move to year 4 using the column method for addition which they were using in Year 3.
Children will build on their knowledge of using Base 10 equipment from Y3, moving to adding 4-digit numbers by the end of Y4 and continuing to use the idea of exchange. Place value counters may be used instead of place value blocks if children are secure in their understanding.

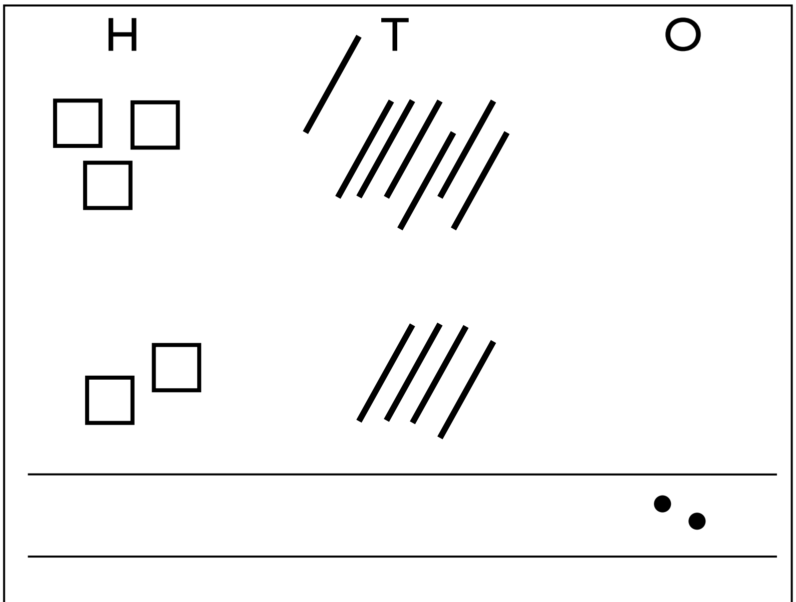
They will use a place value grid to set the calculation out vertically, using Base 10 until confident, and then using jottings, to support their knowledge of exchange between columns (see steps 1-4 below). This will initially be for 3-digit numbers, moving to 4-digit numbers by the end of the year. It is crucial that they continue to use practical methods alongside the written method until they no longer need it. Four digit numbers can be represented by the symbol [Th].

Step 1



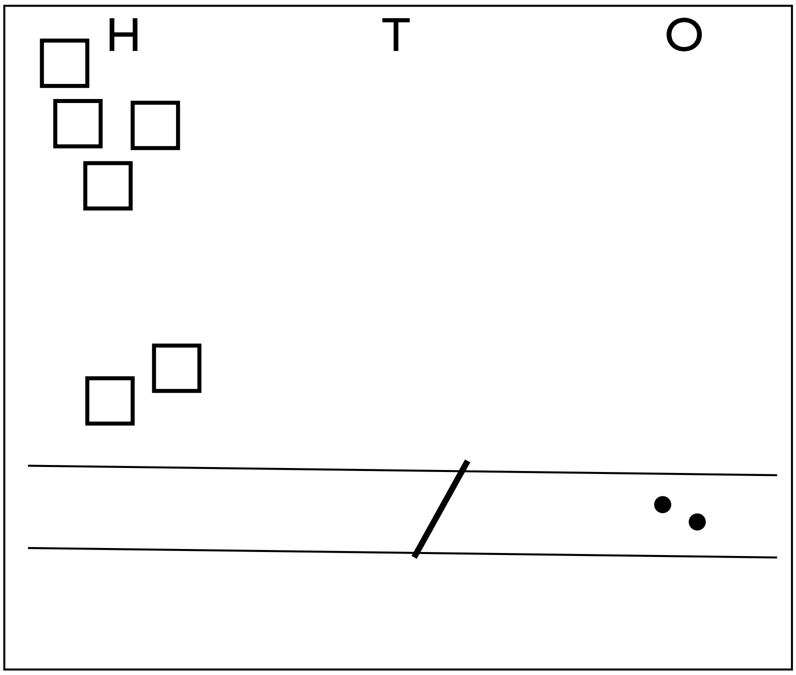
	H	T	O
	3	6	5
+	2	4	7

Step 2



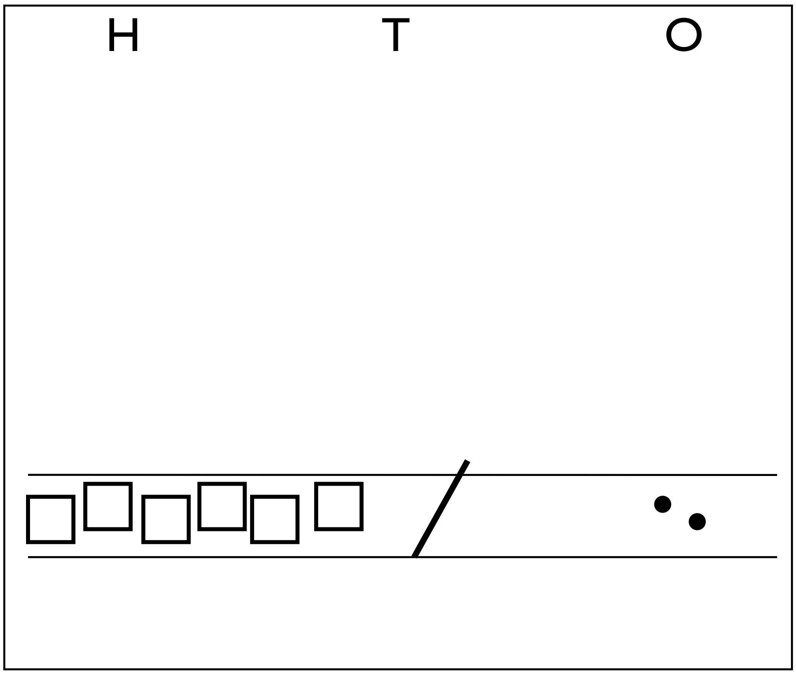
	H	T	O
		1	
	3	6	5
+	2	4	7
			2

Step 3



	H	T	O
	I	I	
	3	6	5
+	2	4	7
		I	2

Step 4



	H	T	O
	I	I	
	3	6	5
+	2	4	7
	6	I	2

By the end of year 4, children should be using the written method confidently and with understanding. They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with one decimal place, knowing that the decimal points line up under one another, however, when adding money children can add up to 2 decimal places.*

Year Five

Children will move to year 5 using the column method for addition which they were using in Year 4.
Children will build on their knowledge of Base 10 equipment from Y4, by using place value counters when adding whole numbers with more than 4-digits by the end of Y5 and continuing to use the idea of exchange.

They will use a place value grid to set the calculation out vertically, using Base 10 or place value counters until confident, and then using jottings, to support their knowledge of exchange between columns. It is crucial that they continue to use practical methods alongside the written method until they no longer need it.

Children should continue to use the carrying method to solve calculations such as:

```
     1 1
   3 3 6 4
 +   2 4 7
 ---------
   3 6 1 1
 ---------
```

```
     1 1
   3 1 2 1
       3 7
 +   1 4 8
 ---------
   3 3 0 6
 ---------
```

```
        1
   3 . 5 6
 + 2 . 4 7
 ---------
   6 . 0 3
 ---------
```

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

Year 6

Children will move to year 6 using the column method for addition which they were using in Year 5.
Children will build on their knowledge from Y5, by using place value counters when adding, if needed.

If required, they will use a place value grid to set the calculation out vertically, using Base 10 or place value counters until confident, to support their knowledge of exchange between columns. It is crucial that they continue to use practical methods alongside the written method until they no longer need it.
Children should extend the carrying method and use it to add whole numbers and decimals with any number of digits.

			1			
	4	0	1	.	2	0
		2	6	.	8	5
+			0	.	7	1
	4	2	8	.	7	6

When adding decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20, using 0 as a placeholder.

They will also be adding:

- several numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another.

Progression Towards a Written Method for Subtraction

In developing a written method for subtraction, it is important that children understand the concept of subtraction, in that it is:

- Removal of an amount from a larger group (take away)
- Comparison of two amounts (difference)

They need to be secure in their understanding of place value, especially partitioning numbers in different ways.

They also need to understand and work with certain principles, i.e. that it is:

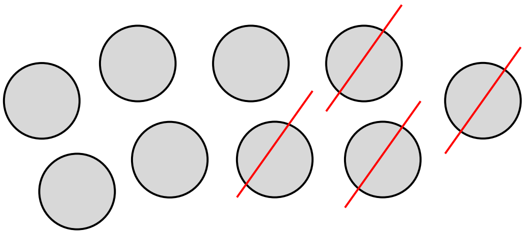
- the inverse of addition
- not commutative i.e. 5 - 3 is not the same as 3 - 5
- not associative i.e. 10 - 3 - 2 is not the same as 10 - (3 - 2)

Reception

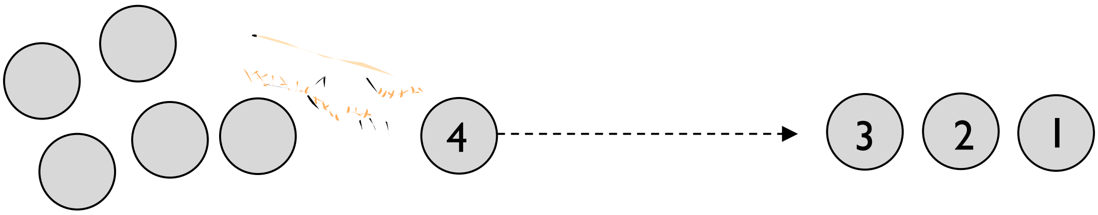
It is very important that children develop a good understanding of the number system, their sense of number and how it works. This should continue to be developed alongside their experiences of subtraction. Practical opportunities should be provided using a wide variety of equipment, including small world play, role play, counters, cubes etc. Key vocabulary should be modelled and used during as many real life situations as possible.

Taking away

Children will begin to develop their ability to subtract by using practical equipment to count out the first number and then remove or take away the second number to find the solution by counting how many are left e.g. 9 – 4.



For illustration purposes, the amount being taken away are shown crossed out. <u>Children would be encouraged to physically remove these using touch counting.</u>



By touch counting and dragging in this way, it allows children to keep track of how many they are removing so they don't have to keep re-counting. They will then touch count the amount that is left to find the answer.

Comparing Difference

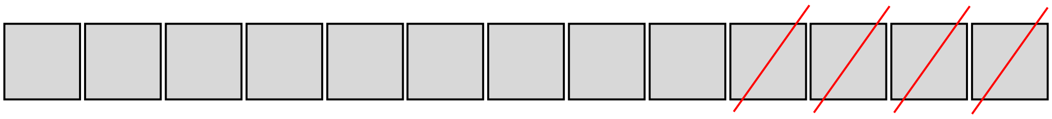
Children should develop their understanding of comparing amounts to find the difference through practical situations e.g. Which tower has more blocks? Are there less/fewer girls than boys?

Those who are ready may record their own calculations.

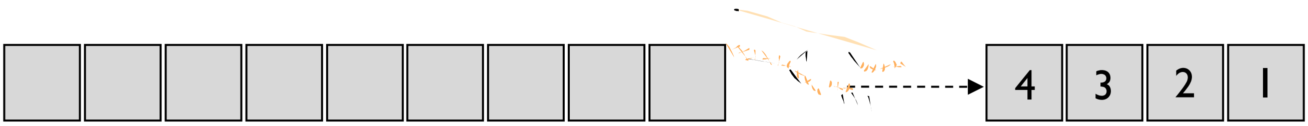
Year One

Taking away

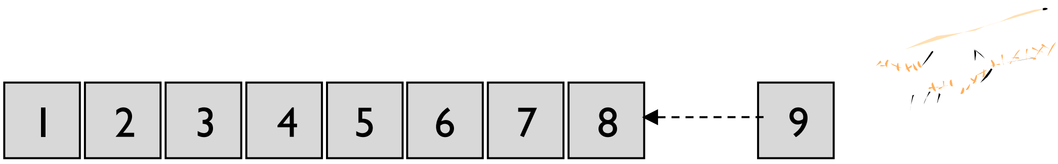
Children will be introduced to formal recording of number sentences e.g 8-5=3. They will continue to use practical equipment and taking away strategies. To avoid the need to exchange for subtraction at this stage, children should continue to use equipment such as counters, cubes and the units from the Base 10 equipment, but not the tens, e.g. 13 - 4



Touch count and remove the number to be taken away, in this case 4.



Touch count to find the number that remains.

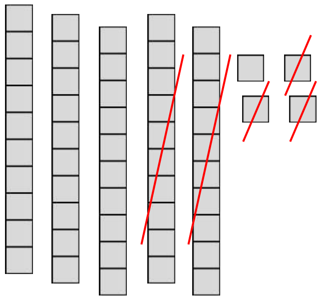


Comparing Difference

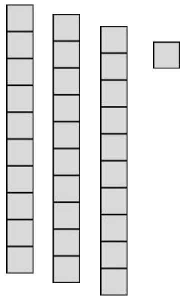
Children should be introduced to the concept of 'difference' practically, using towers of cubes (a physical block diagram) and discussing how we can make one tower the same size as the other. Children's previous work on the relationship between addition and subtraction is crucial in understanding that the difference between 13 and 21 can be written as 21 – 13, but calculated by finding 21 - ? = 13 or that 13 + ? = 21. Numicon can also be used to compare difference between amounts.

Year Two

Children will begin to use straws or Base 10 equipment to support their calculations, still using a take away, or removal, method. They need to understand that the number being subtracted does not appear as an amount on its own, but rather as part of the larger amount. For example, to calculate 54 - 23, children would count out 54 using the Base 10 equipment (5 tens and 4 ones). They need to consider whether there are enough ones to remove 3, in this case there are, so they would remove 3 ones and then two tens, counting up the answer of 3 tens and 1 one to give 31.

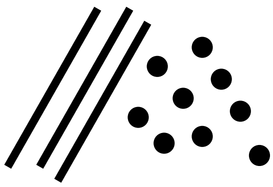


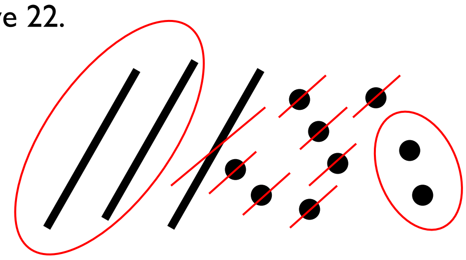
which leaves



so 54 – 23 = 31

Children can also record the calculations using their own jottings of the Base 10 equipment (as slanted lines for the 10 rods and dots for the one blocks), e.g. to calculate 39 – 17 children would draw 39 as 3 tens (lines) and 9 ones (dots) and would cross out 7 ones and then one ten, counting up the answer of 2 tens and 2 ones to give 22.

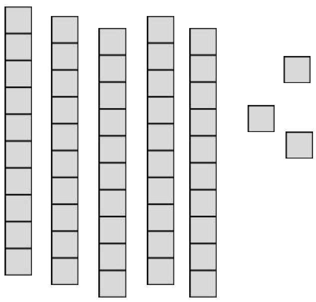




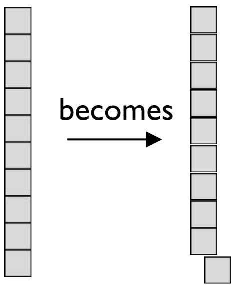
Circling the tens and ones that remain will help children to identify how many remain.

When the amount of ones to be subtracted is greater than the ones in the original number, an exchange method is required. This relies on children's understanding of ten ones being an equivalent amount to one ten. To calculate 53 – 26, by using practical equipment, they would count out 53 using the tens and ones, as in Step 1. They need to consider whether there are enough ones to remove 6. In this case there are not so they need to exchange a ten into ten ones to make sure that there are enough, as in step 2.

Step 1

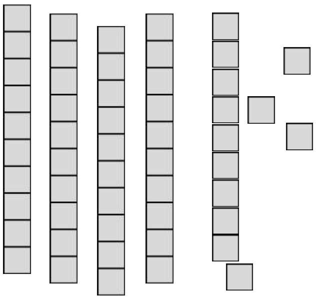


Step 2

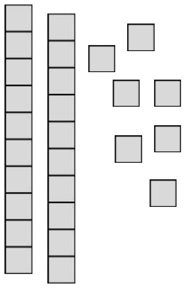


The children can now see the 53 represented as 40 and 13, still the same total, but partitioned in a different way, as in step 3 and can go on to take away the 26 from the calculation to leave 27 remaining, as in Step 4.

Step 3

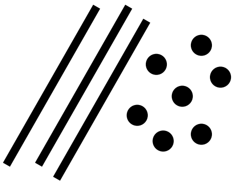


Step 4

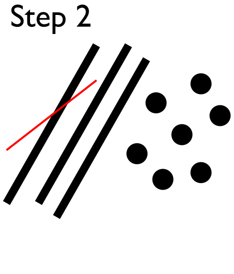


When recording their own jottings, when calculating 37 – 19, children would cross out a ten and exchange for ten ones. Drawing them in a vertical line, as in Step 2, ensures that children create ten ones and do not get them confused with the units that were already in place.

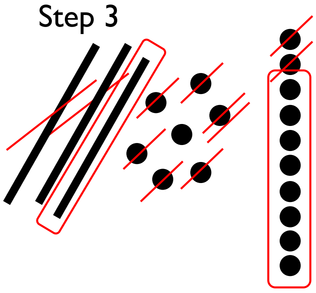
Step 1



Step 2



Step 3



Circling the tens and ones left will help children to identify how many remain.

Comparing Difference

The understanding of difference should be made more secure and the term difference should be used by children. Children need to understand what *how many more* and *how many less/fewer* means and how to solve these types of problems. As in Year One, practical equipment including Numicon should be used.

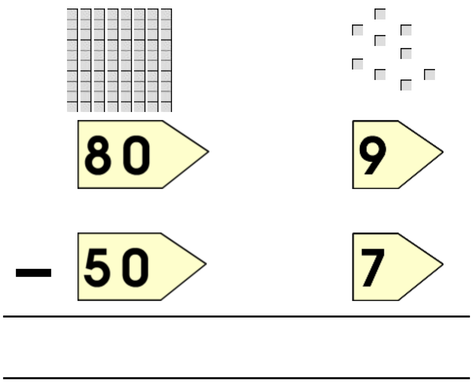
Year Three

Taking away

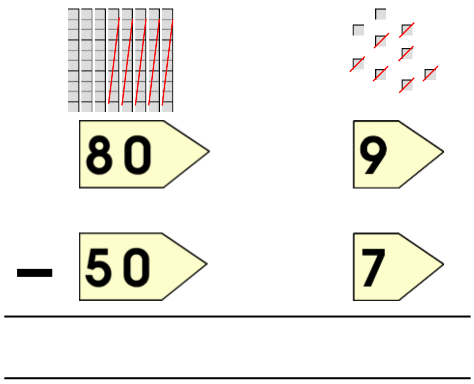
Children will build on their knowledge of using Base 10 equipment from Y2 and continue to use the idea of exchange. This process should be demonstrated using arrow cards to show the partitioning and Base 10 materials to represent the first number, removing the ones and tens as appropriate (as with the more informal method in Y2). It is useful to begin with calculations that don't require exchange before moving to exchanging, then including numbers with 0.

89-57

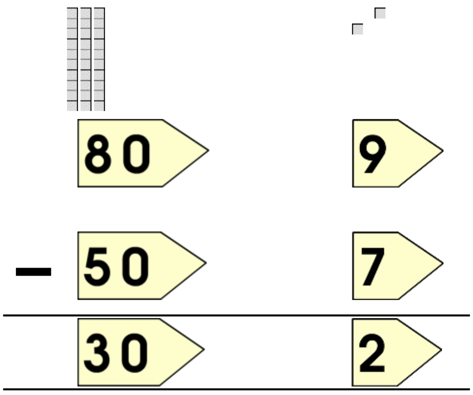
Step 1



Step 2



Step 3



Emphasise that the second (bottom) number is being subtracted from the first (top) number rather than the lesser number from the greater.

This will be recorded by the children as:

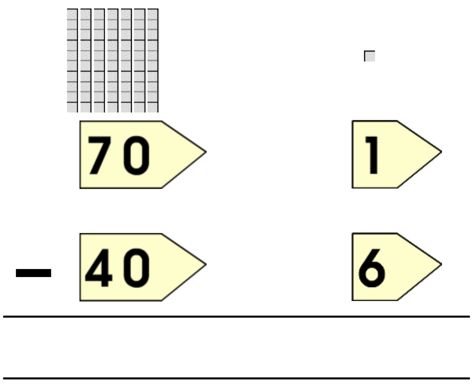
	80	→	9		
-	50	→	7		
	30	→	2	=	32

Children can also use jottings of the Base 10 materials (as in Year 2) to support with their calculation, as in the example below.

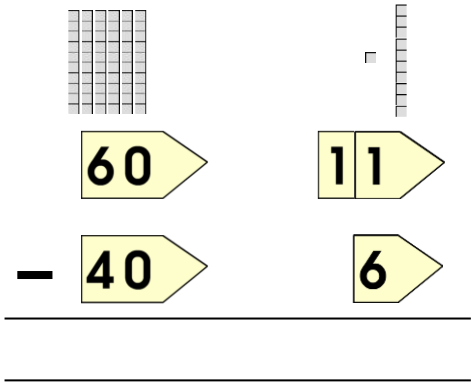
	80	→	9		
-	50	→	7		
	30	→	2	=	32

From this the children will begin to solve problems which involve exchange. Children need to consider whether there are enough ones to remove 6. In this case there is not (Step 1) so they need to exchange a ten into ten ones to make sure that there are enough, as they have been doing in the method for Year 2 (Step 2). They should be able to see that the number is just partitioned in a different way, but the amount remains the same (71 = 70 + 1 = 60 + 11).

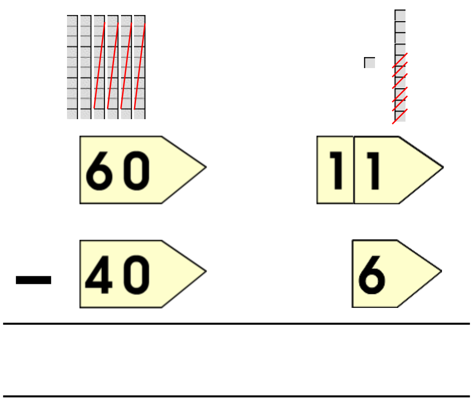
Step 1



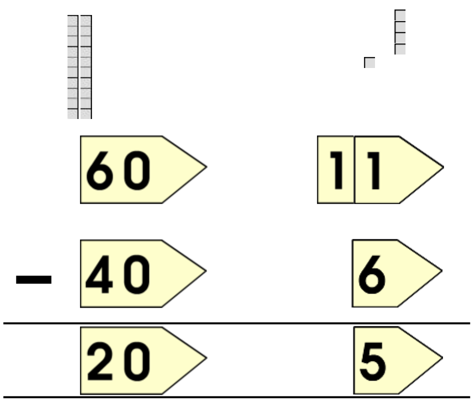
Step 2



Step 3



Step 4



This will be recorded by the children as:

$$
\begin{array}{rcccl}
 & \overset{60}{\cancel{70}} & \rightarrow & {}^{1}1 & \\
- & 40 & \rightarrow & 6 & \\
\hline
 & 20 & \rightarrow & 5 & = \; 25
\end{array}
$$

By the end of year 3, children should also extend this method for three digit numbers.

Comparing Difference

Children should continue to develop their understanding of comparing difference in a range of areas within maths including money, measures and statistics. They should be given opportunities to solve a range of problems and understand that they can use the written method to find the answer if required.

Year Four

Children will move to Y4 using the method they were using as they transitioned from Y3. They will build on their knowledge of using Base-10 for subtraction as they move to working with larger numbers. They will use a place value grid to set the calculation out vertically, using Base 10 until confident, and then using jottings, to support their knowledge of exchange between columns (see steps 1-4 below). This will initially be for 3-digit numbers, moving to 4-digit numbers by the end of the year. It is crucial that they continue to use practical methods alongside the written method until they no longer need it. The calculation will be written alongside the place value grid for children to record a more formal written method for subtraction.

754-286 (Digits representing Base-10 blocks)

Step 1

$$\begin{array}{rrrrrr} & 700 & \rightarrow & 50 & \rightarrow & 4 \\ - & 200 & \rightarrow & 80 & \rightarrow & 6 \\ \hline & & & & & \\ \hline \end{array}$$

Step 2 (exchanging from tens to ones)

$$\begin{array}{rrrrrr} & 700 & \rightarrow & \overset{40}{\cancel{50}} & \rightarrow & {}^{1}4 \\ - & 200 & \rightarrow & 80 & \rightarrow & 6 \\ \hline & & & & & \\ \hline \end{array}$$

Step 3 (exchanging from hundreds to tens)

$$\begin{array}{rrrrrr} & \overset{600}{\cancel{700}} & \rightarrow & \overset{140}{\cancel{50}} & \rightarrow & {}^{1}4 \\ - & 200 & \rightarrow & 80 & \rightarrow & 6 \\ \hline & & & & & \\ \hline \end{array}$$

Step 4

$$\begin{array}{rrrrrrl} & \overset{600}{\cancel{700}} & \rightarrow & \overset{140}{\cancel{50}} & \rightarrow & {}^{1}4 & \\ - & 200 & \rightarrow & 80 & \rightarrow & 6 & \\ \hline & 400 & \rightarrow & 60 & \rightarrow & 8 & = \quad 468 \\ \hline \end{array}$$

This would be recorded by the children as:

$$\begin{array}{rrrrrrl} & \overset{600}{\cancel{700}} & \rightarrow & \overset{140}{\cancel{50}} & \rightarrow & {}^{1}4 & \\ - & 200 & \rightarrow & 80 & \rightarrow & 6 & \\ \hline & 400 & \rightarrow & 60 & \rightarrow & 8 & = \quad 468 \\ \hline \end{array}$$

When children are ready, this leads to the compact method of decomposition (exchanging):

$$\begin{array}{rrrrr} & 4 & \overset{6}{\cancel{7}} & \overset{14}{\cancel{5}} & {}^{1}4 \\ - & 3 & 2 & 8 & 6 \\ \hline & 1 & 4 & 6 & 8 \\ \hline \end{array}$$

By the end of Y4, children should be using the written method confidently and with understanding. Continue to reinforce children's understanding of difference. They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with one decimal place, knowing that the decimal points line up under one another, however, when working out change children will have experience of working with 2 decimal places*

Year Five

Children will move to Y5 using the column method they were using as they transitioned from Y4.

They will build on their knowledge of using Base-10 for subtraction as they move to working with larger numbers.

They will use a place value grid to set the calculation out vertically, using Base 10 until confident, then using drawings, to support their knowledge of exchange between columns. It is crucial that they continue to use practical methods alongside the written method until they no longer need it.

Children should continue to use the decomposition method to solve calculations such as:

$$
\begin{array}{rcccc}
 & \text{Th} & \text{H} & \text{T} & \text{O} \\
 & \overset{6}{\cancel{7}} & {}^{1}0 & \overset{6}{\cancel{7}} & {}^{1}2 \\
- & 3 & 2 & 2 & 6 \\
\hline
 & 3 & 8 & 4 & 6 \\
\hline
\end{array}
\qquad\qquad
\begin{array}{rcccc}
 & \text{O} & . & \text{t} & \text{h} \\
 & \overset{2}{\cancel{3}} & . & \overset{13}{\cancel{4}} & {}^{1}2 \\
- & 1 & . & 7 & 6 \\
\hline
 & 1 & . & 6 & 6 \\
\hline
\end{array}
$$

They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with each number having the same number of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another

Year Six

Children will move to Y6 using the column method they were using as they transitioned from Y5.
They will build on their knowledge of using Base-10 for subtraction as they move to working with larger numbers.
If required, they will use a place value grid to set the calculation out vertically, using Base 10 until confident, then using drawings, to support their knowledge of exchange between columns.
It is crucial that they continue to use practical methods alongside the written method until they no longer need it.

Children should extend the column method and use it to subtract whole numbers and decimals with any number of digits.

$$\begin{array}{rrrrr} & \overset{5}{\cancel{6}} & \overset{13}{\cancel{4}} & {}^{1}3 & 2 \\ - & 4 & 6 & 8 & 1 \\ \hline & 1 & 7 & 5 & 1 \\ \hline \end{array}$$

$$\begin{array}{rrrrrrr} & \overset{3}{4} & {}^{1}\cancel{1} & \overset{6}{7} & . & \overset{11}{2} & {}^{1}\cancel{0} \\ - & & 3 & 4 & . & 7 & 1 \\ \hline & 3 & 8 & 2 & . & 4 & 9 \\ \hline \end{array}$$

When subtracting decimals with different numbers of decimal places, children should be taught and encouraged to make them the same through identification that 2 tenths is the same as 20 hundredths, therefore, 0.2 is the same value as 0.20.

They will also be subtracting:

- numbers with different numbers of digits, understanding the place value;
- *decimals with up to two decimal places (with mixed numbers of decimal places), knowing that the decimal points line up under one another.*
- amounts of money and measures, including those where they have to initially convert from one unit to another.

Progression Towards a Written Method for Multiplication

In developing a written method for multiplication, it is important that children understand the concept of multiplication, in that it is:

- repeated addition

They should also be familiar with the fact that it can be represented as an array

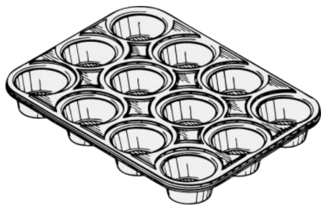
They also need to understand and work with certain principles, i.e. that it is:

- the inverse of division
- commutative i.e. 5 x 3 is the same as 3 x 5
- associative i.e. 2 x 3 x 5 is the same as 2 x (3 x 5)

Reception

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate putting items into resources such as egg boxes, ice cube trays and baking tins which are arrays.



They may develop ways of recording calculations using pictures, etc.

A child's jotting showing the fingers on each hand as a double.

A child's jotting showing double three as three cookies on each plate.

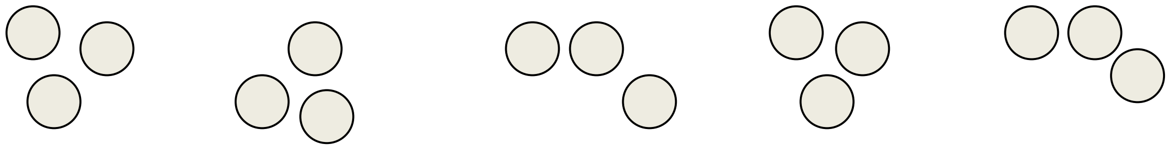
Year One

In Year One, children will continue to solve multiplication problems using practical equipment and jottings. They may use the equipment to make groups of objects. Children should see everyday versions of arrays, e.g. egg boxes, baking trays, ice cube trays, wrapping paper etc and use this in their learning, answering questions such as 'How many eggs would we need to fill the egg box? How do you know?'

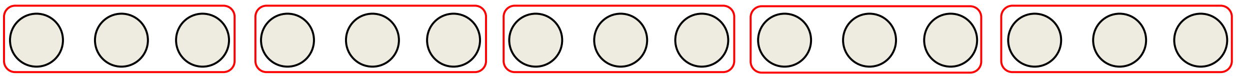
Year Two

Children should understand and be able to calculate multiplication as repeated addition, supported by the use of practical apparatus such as counters or cubes. e.g.

5 x 3 can be shown as five groups of three with counters, either grouped in a random pattern, as below:

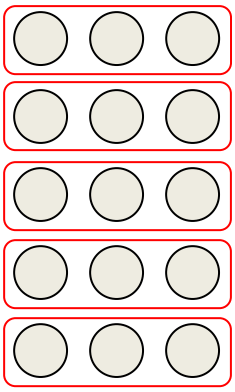


or in a more ordered pattern, with the groups of three indicated by the border outline:

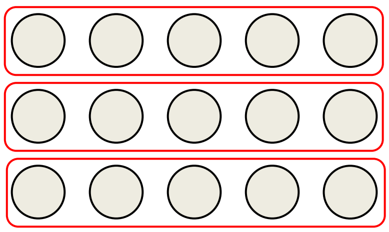


Children should then develop this knowledge to show how multiplication calculations can be represented by an array, (this knowledge will support with the development of the grid method in the future). Again, children should be encouraged to use practical apparatus and jottings to support their understanding, e.g.

5 x 3* can be represented as an array in two forms (as it has commutativity):



3 + 3 + 3 + 3 + 3 = 15



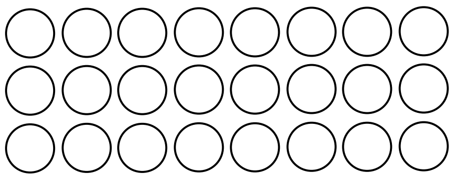
5 + 5 + 5 = 15

*For mathematical accuracy 5 x 3 is represented by the second example above, rather than the first as it is five, three times. However, because we use terms such as 'groups of' or 'lots of', children are more familiar with the initial notation. Once children understand the commutative order of multiplication the order is irrelevant).

Year Three

Initially, children will continue to use arrays where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10),
e.g. 3×8

They may show this using practical equipment; counters or base 10 can be used where appropriate:



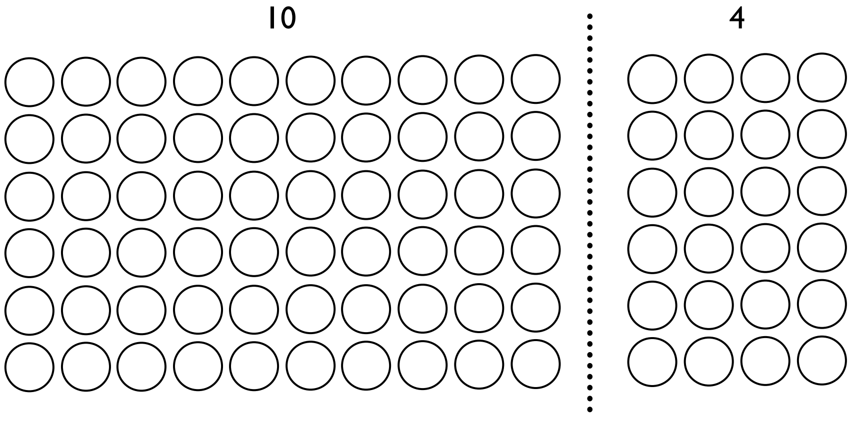
$3 \times 8 = 8 + 8 + 8 = 24$

or by jottings using squared paper:

$3 \times 8 = 8 + 8 + 8 = 24$

	x	x	x	x	x	x	x	x	
	x	x	x	x	x	x	x	x	
	x	x	x	x	x	x	x	x	

As they progress to multiplying a two-digit number by a single digit number, children should use their knowledge of partitioning two digit numbers into tens and ones to help them. For example, when calculating 14×6, children should set out the array, then partition the array so that one array has ten columns and the other four.



Partitioning in this way, allows children to identify that the first array shows 10×6 and the second array shows 4×6.

These can then be added to calculate the answer:

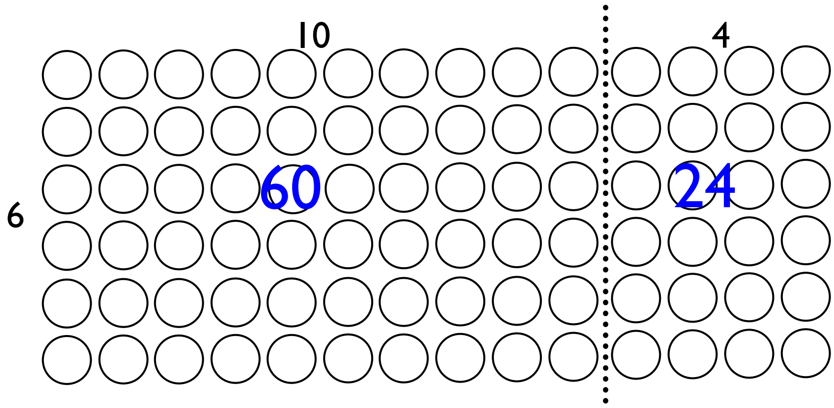
$(6 \times 10) + (6 \times 4)$

$= \quad 60 \quad + \quad 24$

$= \quad 84$

NB There is no requirement for children to record in this way, but it could be used as a jotting to support development if needed.

This method is the precursor step to the grid method. Using a two-digit by single digit array, they can partition as above, identifying the number of rows and the number of columns each side of the partition line.



By placing a box around the array, as in the example below, and by removing the array, the grid method can be seen.

x	10	4
6	60	24

It is very important that children are confident with representing multiplication statements as arrays and understand the rows and columns structure before they develop the written method of recording.

After children have developed a good understanding of the grid method using 2-digit x 1-digit numbers, the written column method can be introduced alongside. Using different colours for each part of the calculation can help children see how the numbers relate to the calculation

Step 1 Record the grid method with written column method alongside.

x	20	4
6		

24
x 6

Step 2 Encourage children to multiply ones first. Record on grid first then children how to record using column method. Use a different colour to show the product of 4 x 6. This helps the children to see the relationship between the grid and written method easily.

x	20	4
6		24

$$\begin{array}{r} 24 \\ \underline{\times\ \ 6} \\ 24 \end{array}$$

Step 3 Multiply tens by the single-digit number. Again record on grid first then children record using column method. Use a different colour to show the product of 20 x 6.

x	20	4
6	120	24

$$\begin{array}{r} 24 \\ \underline{\times\ \ 6} \\ 24 \\ \underline{120} \end{array}$$

Step 4 Add the two totals together and record answer at the bottom of calculation

$$\begin{array}{r} 24 \\ \underline{\times\ \ 6} \\ 24 \\ \underline{120} \\ \underline{144} \end{array}$$

From this, children can use the grid method alongside the written column method to calculate two-digit by one-digit multiplication calculations, initially with two digit numbers less than 20.

Children should also be using this method to solve problems and multiply numbers in the context of money or measures.

Year Four

Children will move to Y4 using whichever method they were using as they transitioned from Y3. They will further develop their knowledge of the grid method to multiply any two-digit by any single-digit number as shown previously in Year 3.

To support this, children should continue to develop their understanding of place value and facts that are linked to their knowledge of tables. For example, children should use their knowledge that 7 x 8 = 56 to know that 70 x 8 = 560.

By the end of the year, they will extend their use of the grid method alongside column method to be able to multiply three-digit numbers by a single digit number.

x	100	30	2
5	500	150	10

$$\begin{array}{r} 132 \\ \times \quad \underline{5} \\ 10 \\ 150 \\ \underline{500} \\ \underline{660} \end{array}$$

Year Five

Children need to revise 2-digit and 3-digit by 1-digit multiplication before moving on.

Once children are confident, they should then be encouraged to multiply a 2-digit number by a 2-digit number using grid method and developing to a formal written method.

Step 1 Record the grid method with written column method alongside.

x	20	1
10		
3		

$$\begin{array}{r} 21 \\ \times\ \underline{13} \end{array}$$

Step 2 Encourage children to multiply 21 by 3 first. Record on grid then show children how to record using column method. Use a different colour to record the products of 1x3 first then 20 x 3.

x	20	1
10		
3	60	3

$$\begin{array}{r} 21 \\ \times\ \underline{13} \\ 63 \end{array}$$

Step 3 Next, multiply 21 by 10. Again record on grid then children record using column method. Use a different colour to record the products of 1x10 and 20x10.

x	20	1
10	200	10
3	60	3

$$\begin{array}{r} 21 \\ \times\ \underline{13} \\ 63 \\ \underline{210} \end{array}$$

Step 4 Add the two totals together and record answer at the bottom of the calculation.

$$\begin{array}{r} 21 \\ \times\ \underline{13} \\ 63 \\ \underline{210} \\ \underline{273} \end{array}$$

Once children are confident multiplying 2-digit by 2-digit numbers, they should then move onto multiplying 3-digit by 2-digit numbers using the grid to support the column method.

Children should continue to use the grid method alongside the written method and extend it to multiplying numbers with up to four digits by a single digit number using column addition to add the totals in the written method.

When children no longer need the grid and have a better understanding of the written method of multiplication they can reduce the amount of recording needed as demonstrated below.

$$\begin{array}{r} 22 \\ \times\ 4 \\ \hline 88 \\ \hline \end{array} \qquad \begin{array}{r} {}^{4}46 \\ \times\ \ 7 \\ \hline 2 \\ \hline \end{array} \quad \begin{array}{r} {}^{4}46 \\ \times\ \ 7 \\ \hline 322 \\ \hline \end{array} \qquad \begin{array}{r} {}^{2}35 \\ \times\ 14 \\ \hline 140 \\ 350 \\ \hline 490 \\ \hline \end{array}$$

Year Six

By Year Six it is hopeful that all children will move from Year 5 using the column method for multiplication. Children will then continue to develop their knowledge of the column method to multiply numbers up to 4-digits by a 2-digit whole number.

They will also move to multiplying one-digit numbers with up to two decimal places by whole numbers using the column method as shown below.

$$\begin{array}{r} {}^{3}2.{}^{1}63 \\ \times \quad 5 \\ \hline 13.15 \\ \hline \end{array}$$

<u>**Progression Towards a Written Method for Division**</u>

In developing a written method for division, it is important that children understand the concept of division, in that it is:

- repeated subtraction

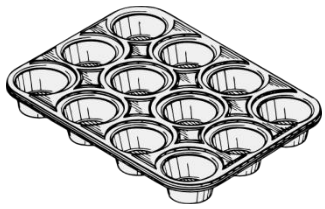
They also need to understand and work with certain principles, i.e. that it is:

- the inverse of multiplication
- not commutative i.e. 15 ÷3 is not the same as 3 ÷ 15
- not associative i.e. 30 ÷ (5 ÷ 2) is not the same as (30 ÷ 5) ÷ 2

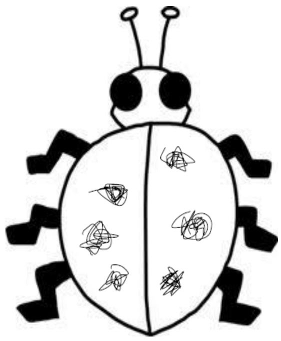
Reception

Children are encouraged to develop a mental picture of the number system in their heads to use for calculation. They should experience practical calculation opportunities using a wide variety of equipment, including small world play, role play, counters, cubes etc.

Children may also investigate sharing items or putting items into groups using items such as egg boxes, ice cube trays and baking tins which are arrays.

They may develop ways of recording calculations using pictures, etc.

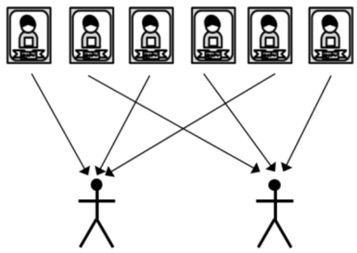


A child's jotting showing halving six spots between two sides of a ladybird.

A child's jotting showing how they shared the apples at snack time between two groups.

Year One

In year one, children will continue to solve division problems using practical equipment and jottings. They should use the equipment to share objects and separate them into groups, answering questions such as 'If we share these six apples between the three of you, how many will you each have? How do you know?' or 'If six football stickers are shared between two people, how many do they each get?' They may solve both of these types of question by using a 'one for you, one for me' strategy until all of the objects have been given out.

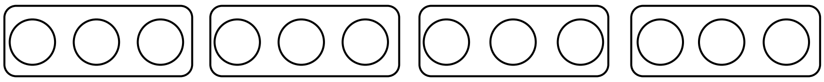


Children should be introduced to the concept of simple remainders in their calculations at this practical stage, being able to identify that the groups are not equal and should refer to the remainder as '… left over'.

Year Two

Children will utilise practical equipment to represent division calculations as grouping (repeated subtraction) and use jottings to support their calculation, e.g.

12 ÷ 3 =

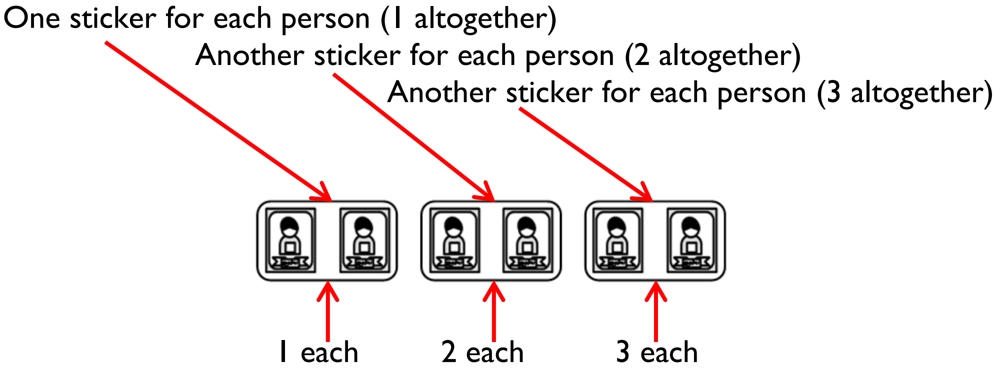


Children need to understand that this calculation reads as 'How many groups of 3 are there in 12?' How many 3s in 12?

The link between sharing and grouping can be modelled in the following way:

To solve the problem 'If six football stickers are shared between two people, how many do they each get?'

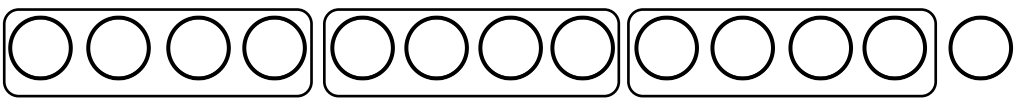
Place the football stickers in a bag or box and ask the children how many stickers would need to be taken out of the box to give each person one sticker each (i.e. 2) and exemplify this by putting the cards in groups of 2 until all cards have been removed from the bag.



Or:

Children should also continue to develop their knowledge of division with remainders, e.g.

13 ÷ 4 =



13 ÷ 4 = 3 remainder 1

Children need to be able to make decisions about what to do with remainders after division and round up or down accordingly. In the calculation 13 ÷ 4, the answer is 3 remainder 1, but whether the answer should be rounded up to 4 or rounded down to 3 depends on the context, as in the examples below:

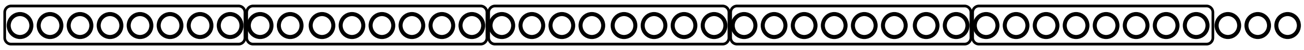
I have £13. Books are £4 each. How many can I buy?
Answer: 3 (the remaining £1 is not enough to buy another book)

Apples are packed into boxes of 4. There are 13 apples. How many boxes are needed?
Answer: 4 (the remaining 1 apple still need to be placed into a box)

Year Three

Initially, children will continue to use division by grouping (including those with remainders), where appropriate linked to the multiplication tables that they know (2, 3, 4, 5, 8 and 10), e.g.

43 ÷ 8 =



43 ÷ 8 = 5 remainder 3

At this stage, children also learn if the remainder should be rounded up or down
e.g. 62÷8=7 remainder 6

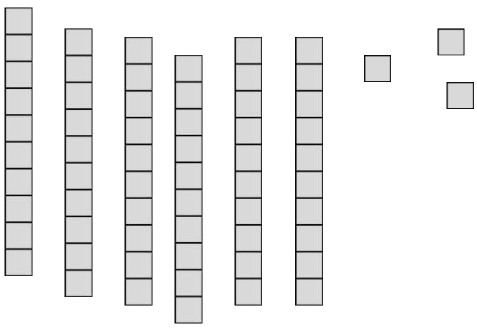
I have 62p. Sweets are 8p each. How many can I buy?
Answer: 7 (the remaining 6p is not enough for another sweet)
Apples are packed into boxes of 8. There are 62 apples. How many boxes do I need?
Answer: 8 (the remaining 6 apples till need to be placed in a box)

For children who are confident with their times table facts, they should then move to short division using place value blocks to develop their understanding.

We want to divide 63 into 3 equal groups. We write

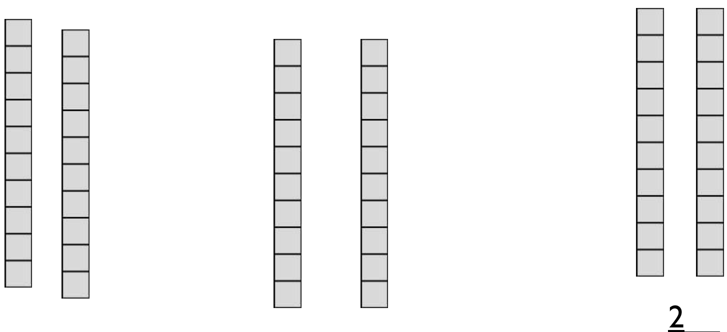
T O
$3\overline{)63}$

How many ten-rods and ones-cubes do I need for 63?



How many groups are we going to share 63 between? 3 groups

First share the tens:



We can put 2 tens in each group. We record like this

$\begin{array}{r}\underline{2}\\ 3)63\end{array}$

Share 3 ones between 3 groups

How many ones in each group? We record it like this $\begin{array}{r} 21 \\ 3\overline{)63} \end{array}$

The answer can be checked using the inverse operation.

Year Four

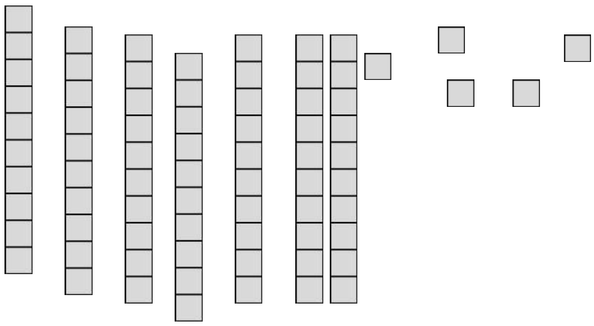
The children should continue to use short division with place value blocks at the beginning including number sentences where they need to regroup, moving to short division without apparatus when dividing numbers up to 3 digits by 1 digit.

Division with regrouping is shown below:

We want to divide 75 into 3 equal groups. We write

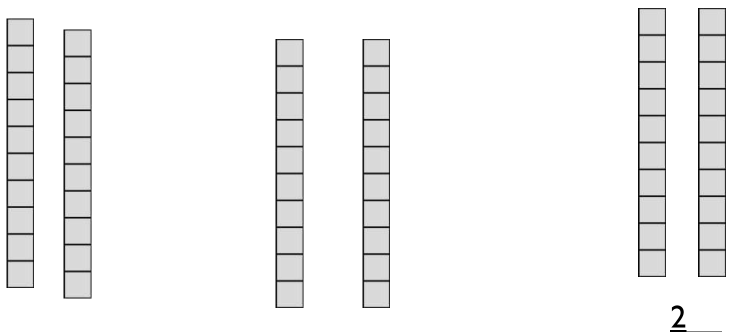
T O
3)75

How many ten-rods and ones-cubes do I need for 75?



How many groups are we going to share 75 between? 3 groups

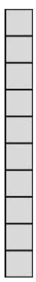
First share the tens:



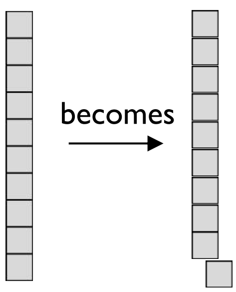
We can put 2 tens in each group. We record like this

 2
3)75

How many tens are left over? 1 ten



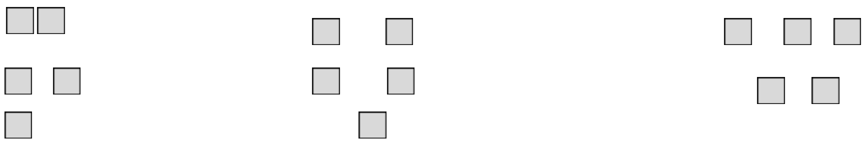
We can't share 1 ten between 3 groups so we need to exchange 1 ten for 10 ones



Add the 10 ones to the 5 ones to make 15 ones

We record it like this: $\begin{array}{l} \underline{2\phantom{^{1}5}} \\ 3)7^{1}5 \end{array}$

Share 15 ones between 3 groups



How many ones in each group? We record it like this $\begin{array}{l} \underline{2^{}5} \\ 3)7^{1}5 \end{array}$

The answer can be checked using the inverse operation.

When children are confident, they should be able to do these without concrete materials.

Year Five

Children should move to dividing up to 4 digits by one digit using short division and expressing the answer as a straightforward remainder *and as a fraction.*

432÷5 becomes

$$\begin{array}{r} \underline{\;\;8\;6\;}\ r2 \\ 5)\,4\,3\,{}^{3}2 \end{array}$$

Answer: 86 remainder 2 or 86 $\frac{2}{5}$

Year Six

Children should continue to use methods they come up with from Year 5.

They should begin to divide up to 4 digits by two digits expressing the answer as a straightforward remainder, as a fraction *and as a decimal (up to 2dp).*

It is very important that children have fluency with their fraction and decimal equivalents.

972÷36

```
      27
36) 196
-    720     20x
     252
-    252      7x
       0
```

Answer: 27

1x	36
2x	72
10x	360
20x	720

432 ÷ 15 becomes

```
          2  8  r 12
1  5 | 4  3  2
       3  0  0
       1  3  2
       1  2  0
          1  2
```

Answer: 28 remainder 12

432 ÷ 15 becomes

```
          2  8
1  5 | 4  3  2
       3  0  0   15×20
       1  3  2
       1  2  0   15×8
          1  2
```

$\frac{\cancel{12}}{\cancel{15}} = \frac{4}{5}$

Answer: 28 $\frac{4}{5}$

432 ÷ 15 becomes

```
          2  8 · 8
1  5 | 4  3  2 · 0
       3  0
       1  3  2
       1  2  0
          1  2  0
          1  2  0
                0
```

Answer: 28·8

Appendix

Key Vocabulary for Addition and Subtraction

add, addition, more, plus, and, make, sum, total, altogether, increase
double, near double
one more, two more, ten more, one hundred more
how many more to make….?
how many more is …. than …..?
how much more is…?
take (away), subtract, subtraction, minus, leave, decrease
how many are left/left over?
how many have gone?
one less, two less, ten less, one hundred less
how many fewer is … than?
how much less is….?
difference between
half, halve
equals, sign, is the same as
tens boundary, hundreds boundary
inverse

Key Vocabulary for Multiplication and Division

lots of, groups of, times, multiply, multiplied by, multiplication
multiple of, product
once, twice, three times, triple, ten times…
times as (big, long, wide etc)
repeated addition
array
row, column
double, halve
share, share equally, divide, division, divided by, divided into
one each, two each, three each…
group in pairs, threes, tens
equal groups of
left, left over, remainder
factor, quotient, divisible by
inverse